Change of variables

Let $F : D \subset \mathbb{R}^2 \to \mathbb{R}^2$ be a C^2 mapping defined in D given by $u = u(x, y), v = v(x, y)$. Let $R \subset D$ be a subregion with smooth boundary $\partial R \subset D$. Then

$$\iint_R (u_x v_y - u_y v_x)\, dxdy = \int_\Gamma u dv\,,$$

where $\Gamma = F(\partial R)$. The proof is based on integration by parts of the left hand side.